STAFFORDSHIRE POLYTECHNIC

DEPARTMENT OF MECHANICAL AND COMPUTER AIDED ENGINEERING

Session 1991/92

BEng(Hons) MECHANICAL ENGINEERING - PART 2
BEng(Hons) COMPUTER AIDED ENGINEERING - PART 2

MATHEMATICS - BM3MH and BM3CH

Date: Wednesday, 29th January, 1992
Time: 9.30 a.m. - 12.30 p.m.
Time allowed 3 hours

Examiners: A. Crossley
D.J. Colwell

ANSWER FIVE QUESTIONS. All questions carry equal marks.

1. The Goodlube company manufactures two grades of machine oil named Normal and Extra which it supplies to wholesalers in 25 litre cans. In the chart below are given (per can) the basic costs, selling prices and inputs of labour and blending chemical.

Product	Labour	Chemical	Production Cost	Selling Price
Normal	3 mins	1 gm	£20	£25
Extra	2 mins	4 gm	£25	£31

On each working day the company has available 15 hours labour and 0.8 kg of the blending chemical. For capacity reasons, daily production of Normal and Extra is limited to 320 and 180 cans respectively. The company wishes to maximise profits. Assuming that the company produces x cans of Extra and y cans of Normal per day, show that the problem can be reduced to the following.

Maximise $P = 6x + 5y$,

subject to
$$4x + y \leq 800 \qquad (1)$$
$$2x + 3y \leq 900 \qquad (2)$$
$$x \leq 180 \qquad (3)$$
$$y \leq 320 \qquad (4)$$

and $x, y \geq 0$.

(5 marks)

Introducing slack variables s_1, s_2, s_3 and s_4 for the respective constraints (1) to (4) construct an initial tableau for the Simplex method of solution.

(2 marks)

/Question 1. continued on page 2....

Question 1. cont'd.

The final tableau is given below.

Basic	P	x	y	s_1	s_2	s_3	s_4	Solution
y	0	0	1	-0.2	0.4	0	0	200
s_3	0	0	0	-0.3	0.1	1	0	30
x	0	1	0	0.3	-0.1	0	0	150
s_4	0	0	0	0.2	-0.4	0	1	120
P	1	0	0	0.8	1.4	0	0	1900

State the problem solution (3 marks)

How would the solution change if an additional 50 gm of chemical became available on each day and an additional hour of labour? (5 marks)

The company is considering the manufacture of a third product called Extra Plus. Each can of Extra Plus would require 3 mins of labour, take 5 gm of chemical, cost £30 to produce and sell for £37. What advice would you give to the company? (5 marks)

2. Four masses in a vibrating system have deflections of x_1, x_2, x_3 and x_4 from their equilibrium positions after the system has been in motion for a time t. The deflections satisfy the differential equations

$$\ddot{x}_1 = -x_1 - x_2 - x_3 - x_4$$
$$\ddot{x}_2 = -x_1 - 2x_2 - 2x_3 - 2x_4$$
$$\ddot{x}_3 = -x_1 - 2x_2 - 3x_3 - 3x_4$$
$$\ddot{x}_4 = -x_1 - 2x_2 - 3x_3 - 4x_4$$

(i) Using trial solutions $x_1 = X_1 \sin\omega t$, $x_2 = X_2 \sin\omega t$, $x_3 = X_3 \sin\omega t$ and $x_4 = X_4 \sin\omega t$ show that ω^2 is an eigenvalue and $(X_1, X_2, X_3, X_4)^T$ an eigenvector of the matrix

$$A = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 2 & 2 \\ 1 & 2 & 3 & 3 \\ 1 & 2 & 3 & 4 \end{pmatrix}.$$

(6 marks)

(ii) Show that if λ is an eigenvalue of a matrix A then λ^2 is an eigenvalue of A^2. (4 marks)

(iii) Given that $A^2 = \begin{pmatrix} 4 & 7 & 9 & 10 \\ 7 & 13 & 17 & 19 \\ 9 & 17 & 22 & 26 \\ 10 & 19 & 26 & 30 \end{pmatrix}$

use an iterative method with A^2 to find the dominant eigenvalue for A and its corresponding eigenvector. Work with three decimal places until the eigenvector components are consistent to two decimal places. (10 marks)

3.(a) At time t, the temperature u(t) of a cooling object is given by the equation

$$\frac{du}{dt} + 0.1u = 2.5(2-t), \quad u(0) = 80.$$

Use the modified Euler method, correcting once at each step, to obtain estimates of u when t = 0.1 and t = 0.2. (Work to three decimal places.)

(10 marks)

(b) Use central difference approximations to reduce the boundary value problem

$$x\frac{d^2y}{dx^2} - 2\frac{dy}{dx} = 4, \quad y(0) = y(1) = 0,$$

to a difference equation at the general point (x_i, y_i).

Hence obtain a set of linear equations satisfied by estimates of $y(x)$ at $x = \frac{1}{4}, \frac{1}{2}, \frac{3}{4}$ and solve these equations.

(10 marks)

4. A periodic waveform, period 6, may be modelled by

$$f(t) = \begin{cases} -2, & -3 < t < -1 \\ 0, & -1 < t < 1 \\ 2, & 1 < t < 3. \end{cases}$$

(i) Sketch the waveform for $-9 < t < 9$. (2 marks)

(ii) Obtain the whole-range Fourier coefficients of the waveform and list the first three non-zero terms in its Fourier series. (8 marks)

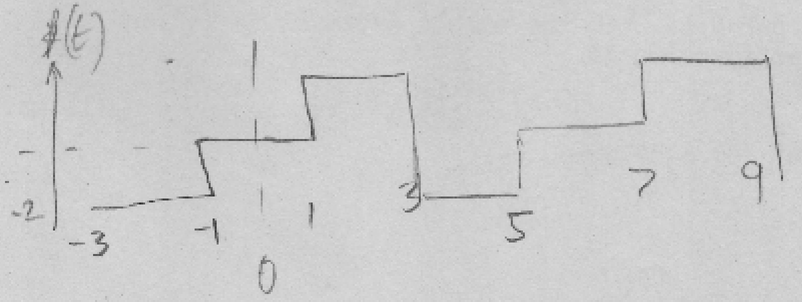
(iii) Sketch the amplitude frequency spectrum as far as the fourth harmonic. (2 marks)

(iv) Suppose now that a distorted version of the periodic waveform is represented by the following sampled values.

t	-3	-2	-1	0	1	2	3
f(t)	-0.9	-2.0	-0.5	0	0.6	2.4	1.1

Using a rectangular approximation, obtain estimates for the d.c term and the first harmonic coefficients in the Fourier series of the waveform.

(8 marks)

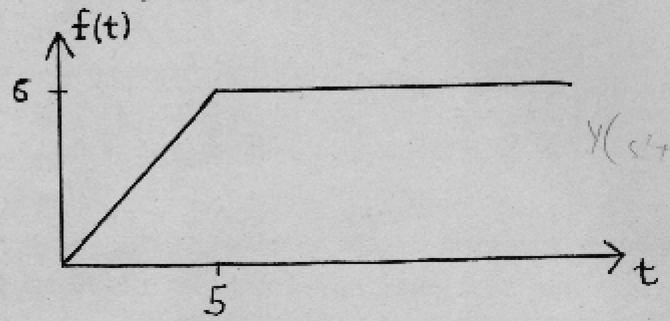


odd $a_n, a_0 = 0$

5.(a) Obtain the inverse Laplace transform of each of the transfer functions

(i) $\dfrac{2s + 5}{(s+3)(s+4)}$, (ii) $\dfrac{3s + 5}{s^2+8s+20}$. (6 marks)

(b) Express the signal f(t), shown graphically in the figure, in terms of unit step functions. Hence, or otherwise, obtain the Laplace transform of the signal.



(6 marks)

(c) A system may be modelled by the initial value problem

$$2\frac{d^2y}{dt^2} + 5\frac{dy}{dt} + 2y = h(t),$$

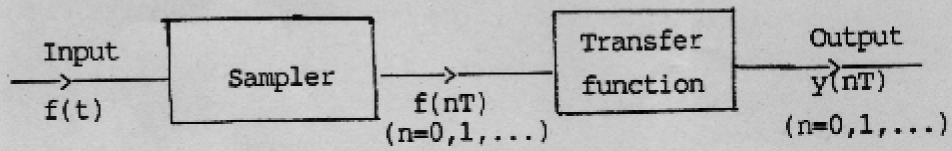
$$y(0) = 0, \quad y'(0) = 2.$$

(i) Obtain the system transfer function.

(ii) Obtain the system response y when the input is $h(t) = 3e^{-3t}$.

(8 marks)

6.(a) A system may be modelled by the following block diagram in which the sampler operates with a sample interval T=2.



Obtain the output from the system when the transfer function is

$$\frac{z + \frac{2}{9}}{z + \frac{1}{3}}$$

and the input function is $f(t) = 4(\frac{1}{3})^t$. (9 marks)

(b) The output from a system is y(t), where t=nT (n=0,1,...) and, due to the presence of a feedback loop, the total input to the system is $f(t) + \frac{1}{5}y(t-T)$, where

$$f(t) = 3(\tfrac{1}{2})^t.$$

Obtain the output y(t) when the sample interval is T=1 and the system transfer function is

$$G(z) = \frac{z}{z-\frac{1}{4}}.$$

(11 marks)

$(2s^2 + 5s + 2)$

$= (2s+1)(s+2)$

$= 2s^2 + 4s + s + 2$

7. A machine maintenance company keeps records of the length of time it takes to repair a faulty machine. The time that elapses between a customer reporting a machine out of action due to a fault and the fault being repaired is known as the "repair time". The company's records show that repair times may be assumed to be normally distributed with mean 40 hours and standard deviation 5 hours and that a repair time on any particular occasion that a fault is reported may be assumed to be independent of all other repair times.

(i) Find the probability that on a particular occasion the repair time will be less than 24 hours. (4 marks)

(ii) The company wishes to state in its advertisements that 99% of all repair times will be less than a certain number n of hours. Find the smallest number of hours that would truthfully be used in the advertisements. (5 marks)

(iii) Find the probability that the sum of the repair times on two separate occasions, chosen at random, will be more than 100 hours. (5 marks)

(iv) If a repair time is exceptionally long, the maintenance company pays compensation to the customer. The rate of compensation is £100 if the repair time is between 50 and 55 hours and £250 if the repair time exceeds 55 hours. Calculate the expected amount of compensation per fault. (6 marks)

8.(a) A company inspects 1000 articles on each day during four weeks in October. The numbers of defectives recorded are shown below.

Day	Number of Defectives	Proportion Defective P
01	62	0.06
02	81	0.08
03	74	0.07
04	121	0.12
05	120	0.12
08	60	0.06
09	131	0.13
10	55	0.06
11	72	0.07
12	61	0.06
15	56	0.06
16	70	0.07
17	55	0.06
18	69	0.07
19	58	0.06
22	65	0.07
23	66	0.07
24	55	0.06
25	57	0.06
26	60	0.06
TOTAL	1448	

/Question 8.(a) continued on page 6....

Question 8.(a) cont'd.

Estimate the central value and control limits for this situation and construct a control chart from the given data showing the results for the twenty days.

Comment on the chart and give views (without calculations) on the strategy that you would adopt for controlling production over the next four weeks. (10 marks)

8.(b) To control the production of steel rods samples of five rods are taken at regular time intervals to have their diameters measured. The sample mean $\bar{x}$ and the range R obtained from each of the first 10 samples are given below.

Sample	1	2	3	4	5
$\bar{x}$ mm	2.480	2.455	2.492	2.473	2.495
R mm	0.12	0.10	0.09	0.09	0.08

Sample	6	7	8	9	10
$\bar{x}$ mm	2.520	2.495	2.510	2.543	2.524
R mm	0.08	0.05	0.02	0.10	0.03

(i) Estimate the mean μ and the standard deviation σ of the diameter of the rods produced by the machine. (2 marks)

(ii) Find warning and action limits for the mean diameter and for the range of the diameters. (2 marks)

(iii) Is the mean capable of meeting rod diameter tolerance limits of 2.5 ± 0.1 mm? (2 marks)

(iv) The next five samples give the following results.

Sample	11	12	13	14	15
$\bar{x}$ mm	2.510	2.490	2.495	2.515	2.500
R mm	0.11	0.13	0.140	0.160	0.175

What might be happening? (2 marks)

(v) The next five samples give the following results.

Sample	16	17	18	19	20
$\bar{x}$ mm	2.500	2.480	2.471	2.456	2.440
R mm	0.08	0.10	0.09	0.11	0.10

Suggest an explanation. (2 marks)