STAFFORDSHIRE UNIVERSITY
School of Engineering

Staffordshire
UNIVERSITY

Session 1992/93

BEng(Hons) in Computer Aided Engineering (BM4CH)

Computer Aided Design

DATE: 7th June 1993

TIME: 2.00 pm - 5.00 pm

1,2,4,5,7

DURATION: 3 Hours

EXAMINER:	D G Cheshire	**Ext No:**	5324
	C Buckingham		5271
	B Delves		5266

Instructions to Candidates:

This paper contains eight questions.
Candidates should answer five questions only.

Students Will Require:

1. (a) The parametric definition of a single cubic spline segment can be expressed in matrix form in terms of a parameter t, the two endpoints P_0 and P_1 and the slope at the two endpoints P_0' and P_1' as follows:-

$$P(t) = [T][N][G]$$

$$P(t) = \begin{bmatrix} 1 & t & t^2 & t^3 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -3 & 3 & -2 & -1 \\ 2 & -2 & 1 & 1 \end{bmatrix} \begin{bmatrix} P_0 \\ P_1 \\ P_0' \\ P_1' \end{bmatrix}$$

Using this information and referring to Figure Q1, which shows a cubic B-Spline curve segment and its controlling polygon, show that the definition of a B-Spline segment is given by:-

$$P(t) = [T][N][G]$$

$$P(t) = \begin{bmatrix} 1 & t & t^2 & t^3 \end{bmatrix} \frac{1}{6} \begin{bmatrix} 1 & 4 & 1 & 0 \\ -3 & 0 & 3 & 0 \\ 3 & -6 & 3 & 0 \\ -1 & 3 & -3 & 1 \end{bmatrix} \begin{bmatrix} V_0 \\ V_1 \\ V_2 \\ V_3 \end{bmatrix}$$

(8 marks)

(b) Plot 6 equi-spaced points of the two dimensional B-Spline segment whose controlling vertices are:-

V_0 = (1,1)
V_1 = (2,3.5)
V_2 = (4,4)
V_3 = (5,2)

(6 marks)

(c) B-Spline cubic curves exhibit third order continuity at knot points. Explain this statement.

(6 marks)

2. (a) Many CAD systems use piece-wise cubic polynomial splines to define freeform curves.

(i) What are the advantages of using cubic, as opposed to higher order, polynomials?

(ii) What are piece-wise splines and why are they used?

(4 marks)

(b) Each side of the boat hull shown in Figure Q2 is to be defined by a 2 segment Ferguson piece-wise polynomial curve with a knot point at the waterline. The following formula can be used to derive the intermediate tangent vectors for such a curve:-

$$\begin{bmatrix} 1 & 0 & 0 \\ 1 & 4 & 1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} P_1' \\ P_2' \\ P_3' \end{bmatrix} = \begin{bmatrix} P_1' \\ 3(P_3 - P_1) \\ P_3' \end{bmatrix}$$

What would influence your choice of suitable end tangent vectors P_1' and P_3'?

(4 marks)

(c) Using the values (0,-3) for P1′ and (0.521,-2.954) for P3′ and assuming the origin to be located at the base of the hull, calculate the intermediate tangency vector P2′ for the left hand side of the hull. Use these to calculate the position of the <u>parametric midpoints</u> of each of the segments. Sketch a graph of these points showing the shape of the whole curve.

(12 marks)

3. (a) Surface modelling CAD systems use a variety of mathematical bases for the definition of curves from which surfaces are derived. When selecting a CAD system the purchaser should be aware of the effects that the underlying mathematics has on the operation of the system. In this light discuss the advantages and disadvantages of each of the following four popular mathematical bases.

(i) Ferguson Cubic

(ii) Bezier

(iii) B-Splines

(iv) Non-Uniform Rational B-Spline

(8 marks)

(b) Derive a 3 dimensional parametric equation to describe an elliptical helix of pitch P whose axis lies coincident with the y axis. The major radius of the helix, which lies in the direction of the x axis, should be denoted by r_1 and the minor axis, which lies in the direction of the z axis, should be denoted by r_2. The helix should pass through the point $(r_1,0,0)$.

(8 marks)

(c) Explain the terms explicit curves and implicit curves. Discuss the advantages and disadvantages of each.

(4 marks)

4. A beam is built in at both ends and carries a distributed load along its entire span as shown in Figure Q4.

Use a finite element model consisting of two elements in order to obtain :-

(i) The mid-span deflection.

(12 marks)

(ii) The bending moment diagram.

(8 marks)

The stiffness matrix for a beam element is given as :-

$$[k^e] = \frac{EI}{L^3}\begin{bmatrix} 12 & 6L & -12 & 6L \\ 6L & 4L^2 & -6L & 2L^2 \\ -12 & -6L & 12 & -6L \\ 6L & 2L^2 & -6L & 4L^2 \end{bmatrix}$$

The consistent load vector for a beam element is given as :-

$$\{P^e\} = \begin{Bmatrix} \frac{pL}{2} \\ \frac{pL^2}{12} \\ \frac{pL}{2} \\ \frac{-pL^2}{12} \end{Bmatrix}$$

5. This question relates to a bar element of uniform cross-section as shown in Figure Q5. The element has nodal axial displacement degrees of freedom u_1 and u_2.

(i) Part of the process of forming the element stiffness matrix is to represent the displacement at any point on the element in terms of the nodal displacements. If the displacement is u at a distance x from node 1, this relationship takes the form

$$\{u\} = [N_1 \;\; N_2]\begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix}$$

Derive the shape functions N_1 and N_2 directly and <u>without</u> commencing from the assumed displacement function for the element.

(7 marks)

(ii) Why is the approach used in (i) above to be preferred to commencing the formulation with the assumed displacement function? Illustrate your answer in the context of this particular element.

(7 marks)

(iii) For this particular element, the strain vector $\{\varepsilon\} = \varepsilon_x = du/dx$ and the Elasticity matrix [D] is given as simply the Modulus of Elasticity, E. Make use of the expression below in order to derive the stiffness matrix for the element.

$$[k^e] = \int_V [B]^t \, [D] \, [B] \, dvol$$

where [B] relates the strain to the nodal displacements.

(6 marks)

6. The beam shown in Figure Q6 is to be analysed using the Rayleigh method.

(i) Use a general fourth order polynomial of the form

$$y = a + bx + cx^2 + dx^3 + ex^4$$

to model the assumed displacement and adapt it to satisfy both the geometric and static boundary conditions.

(6 marks)

(ii) Use the assumed displacement function developed above in order to derive an expression for the deflection at the load point.

(8 marks)

(iii) Describe how the Rayleigh approach used above may be extended into the Rayleigh-Ritz method and indicate the advantages that may accrue.

(3 marks)

(iv) Discuss the limitations of the Rayleigh-Ritz approach and indicate how the technique is developed into the finite element method.

(3 marks)

7. (a) Why is the selection of an appropriate computer representation of a human expert's knowledge so important and what are some desirable qualities of it? Illustrate the answer with reference to an example problem.

(10 marks)

(b) Describe the basic architecture of expert systems.

(6 marks)

(c) Why is the explanation facility central to the success of an expert system and how do rules make the generation of simple explanations easy?

(4 marks)

8. The following Prolog clauses represent a tiny animal identification program:

```
has( tigger, hair).
has( tigger, pointed-teeth).
has( tigger, claws).
has( tigger, black-stripes).

instance( Animal, cheetah) :-
        isa( Animal, carnivore),
        has( Animal, dark-spots).

instance( Animal, tiger) :-
        isa( Animal, carnivore),
        has( Animal, black-stripes).

isa( Animal, mammal) :-
        has( Animal, milk).

isa( Animal, mammal) :-
        has( Animal, hair).

isa( Animal, carnivore) :-
        isa( Animal, mammal),
        has( Animal, pointed-teeth),
        has( Animal, claws).
```

(i) Give a detailed explanation with appropriate diagrams of how the Prolog interpreter would answer the following query:

```
?- instance( tigger, What).
```

You will need to generate the proof tree (alternatively known as a goal or and/or tree) and make it clear which goals succeed and fail.

(12 marks)

(ii) What is the difference between a backward-chaining system like Prolog and one which uses forward chaining?

(4 marks)

(iii) What are the implications of the two types of chaining both in obtaining facts from the user and for generating explanations?

(4 marks)

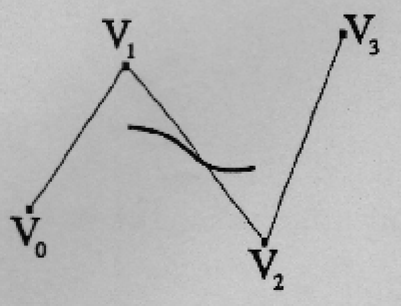

Figure Q1 - A B-Spline Segment

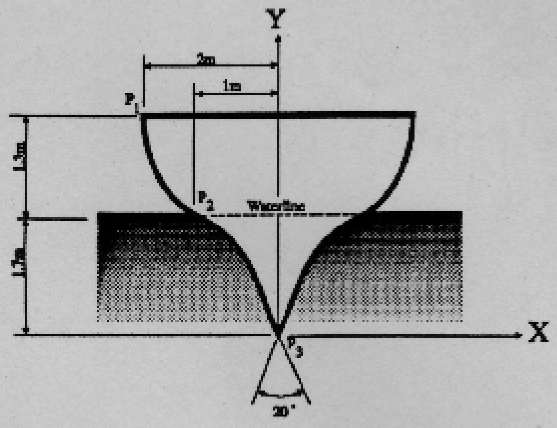

Figure Q2 - A Boat Hull

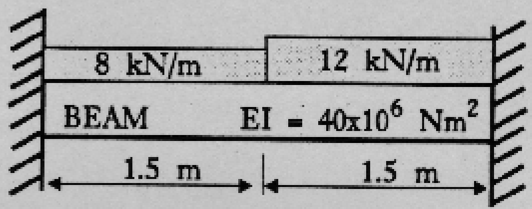

Figure Q4

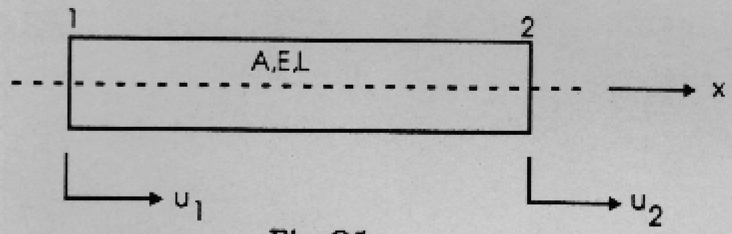

Fig Q5

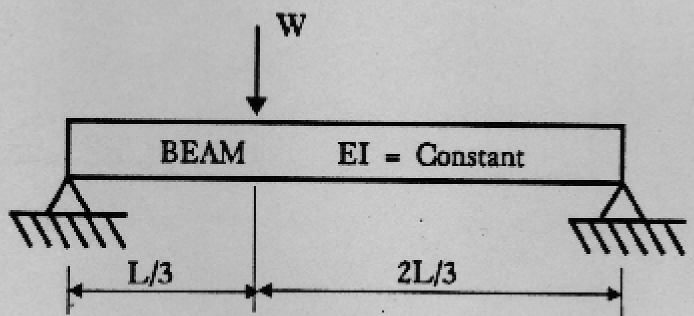

Figure Q6

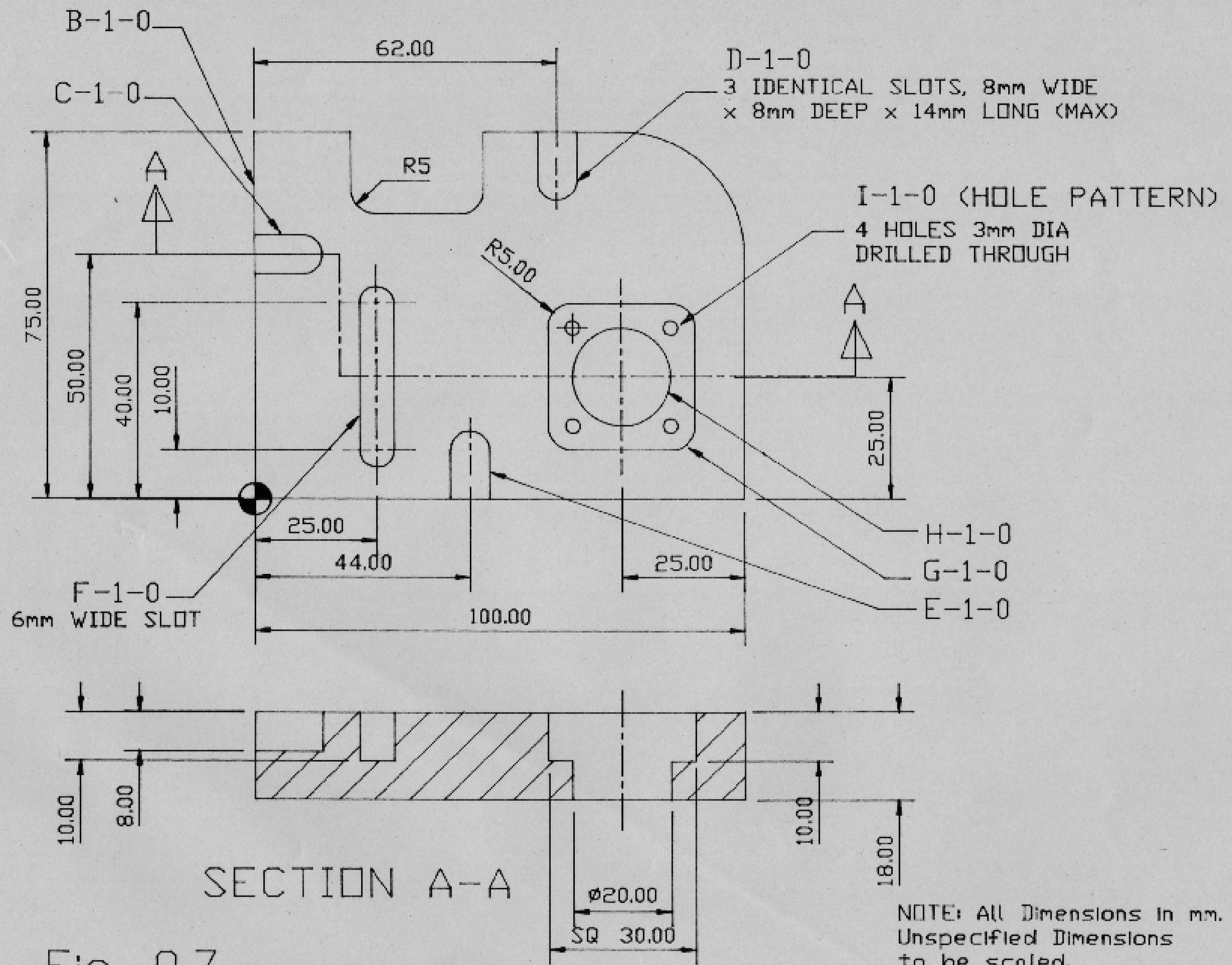

Fig. Q.7

NOTE: All Dimensions in mm.
Unspecified Dimensions
to be scaled.